Basic topology 3

Analyze which of the following sets are compact.
In $\mathbb{R}$:

1. $A = [-7, 1] \cup [5, 13]$
2. $B = (-7, 1) \cup (5, 13)$
3. $C = [-3, -1] \cup (2, +\infty)$
4. $D = [1, 5] \cup [7, 9]$
5. $E = \{1, 2, 3, 4, 5\}$
   Sets in $\mathbb{R}^2$
6. $A = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$
7. $B = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$
8. $C = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$

Solutions

For a set to be compact, it must be closed and bounded.

Sets 1, 2, 4, and 5 are bounded, because each of these sets can be contained within a specific interval. Set 3 is not bounded.

Set 1 is closed because around every point not in the set, we can find a ball that does not intersect with the set. Set 2 is not closed because for any point that is an endpoint of one of the open intervals, every ball around it will intersect with the set. Set 3 is not closed because for any ball around a point just larger than 2, there will be an intersection with the set, and no ball can be drawn around $+\infty$ that lies completely outside of the set.

Set 4 is closed because around every point not in the set, we can find a ball that does not intersect with the set. Set 5 is closed because it consists of isolated points, and around each point not in the set, we can find a ball that does not intersect with the set.

Sets 1, 4 and 5 are compact because they are both closed and bounded. Set 2 is not compact because it is not closed, even though it is bounded. Set 3 is not compact because it is not bounded, even though it is closed in its bounded interval portion.

6 7 and 8 are all bounded, since we can take a ball with a radius greater than any of them contains. For example, if we take $r = 2$, those sets are included in $B((0,0),2)$. Concerning being closed, only the sets 7 and 8 are. **Therefore, 7 and 8 are compact sets.**